

Appendix

The correlation coefficients r_{P_1, P_2} and $r_{T, \frac{P_1 + P_2}{2}}$ are related, especially when the correlation across all treatment arms is the same, i.e., $r = r_{T, P} = r_{T, P_1} = r_{T, P_2} = r_{P_1, P_2} = \frac{\sigma_B^2}{\sigma_B^2 + \sigma_W^2}$.

$$\begin{aligned}
 r_{T, \frac{P_1 + P_2}{2}} &= \frac{\text{cov}\left(y_T, \frac{y_{P_1} + y_{P_2}}{2}\right)}{\sqrt{\text{var}(y_T) \text{var}\left(\frac{y_{P_1} + y_{P_2}}{2}\right)}} \\
 &= \frac{\text{cov}(y_T, y_{P_1})}{\frac{1}{2} \sqrt{\text{var}(y_T) [2 \text{var}(y_{P_1}) + 2 \text{cov}(y_{P_1}, y_{P_2})]}} \\
 &= \frac{\sqrt{\text{var}(y_T) \text{var}(y_{P_1}) \text{cor}(y_T, y_{P_1})}}{\frac{1}{\sqrt{2}} \sqrt{\text{var}(y_T) \text{var}(y_{P_1}) [1 + \text{cor}(y_{P_1}, y_{P_2})]}} \\
 &= \frac{\text{cor}(y_T, y_{P_1})}{\frac{1}{\sqrt{2}} \sqrt{[1 + \text{cor}(y_{P_1}, y_{P_2})]}} \\
 &= \frac{r_{T, P_1}}{\sqrt{(1 + r_{P_1, P_2})/2}}
 \end{aligned}$$

Hence

$$\begin{aligned}
 \frac{\sigma_5^2}{\sigma_4^2} &= \frac{1 + \frac{(1 + r_{P_1, P_2})}{2} - 2 r_{T, \frac{P_1 + P_2}{2}} \sqrt{\frac{(1 + r_{P_1, P_2})}{2}}}{2(1 + r_{T, P})} \\
 &= \frac{1 + \frac{(1 + r_{P_1, P_2})}{2} - 2 r_{T, P_1}}{2(1 + r_{T, P})} \\
 &= \frac{\frac{3(1 + r_{T, P_1})}{2}}{2(1 + r_{T, P})} \\
 &= \frac{3}{4}
 \end{aligned}$$